Title: Roth's theorem on arithmetic progressions

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Abstract: In the presented summary work we study sets of natural numbers not containing arithmetic progressions. The aim of this thesis is to provide an overview and comparison of both analytical and combinatorial proofs of Roth's theorem, which states that every set of positive upper asymptotic density contains arithmetic progression of length three. We also focus on the Erdős-Turán conjecture and Szemerédi's theorem, which finally settled the conjecture for arithmetic progressions of arbitrary length k. In the end, we introduce the bounds for the number $r_3(n)$, which corresponds to the largest size of a subset $A \subseteq [n]$, which contains no arithmetic progressions of length three. At the end we present two constructions of progression-free sets.

Keywords: Additive number theory, Arithmetic progressions, Roth's theorem, Elkin's construction